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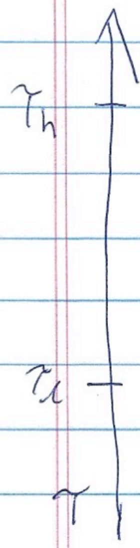
8.1(a). Entropy. Heat ~~Work~~



$\sigma_h = Q_h/\tau_h$ $\qquad Q_h = Q_l + W$

$\sigma_l = Q_l/\tau_l$ $\qquad Q_l.$

Impose $\sigma_h = \sigma_l \Rightarrow Q_h/\tau_h = Q_l/\tau_l.$

$$W = Q_h - Q_l = Q_h - Q_h \frac{\tau_l}{\tau_h}.$$

$$\Rightarrow \frac{W}{Q_h} = \boxed{1 - \frac{\tau_l}{\tau_h}}$$

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8.1(b). Carnot engines are known to have efficiency $1 - \frac{\tau_l}{\tau_{hh}}$.

$\Rightarrow \quad W = Q_{hh}\left[1 - \frac{\tau_l}{\tau_{hh}}\right]$, or $1 - \frac{\tau_l}{\tau_{hh}}$ is the amount of work available per unit of heat at τ_{hh} is consumed. The amount of heat from the heat pump at τ_h per unit of work consumed is given by $\frac{Q_h}{W} = \frac{\tau_h}{\tau_h - \tau_l}$.

In essence, we are "converting" heat at τ_{hh} to heat at τ_h. The overall efficiency is given by

$$\frac{Q_h}{Q_{hh}} = \frac{Q_h}{W}\frac{W}{Q_{hh}}$$

$$= \left(\frac{\tau_h}{\tau_h - \tau_l}\right)\left(1 - \frac{\tau_l}{\tau_{hh}}\right),$$

$$\text{or} \quad \frac{Q_{hh}}{Q_h} = \left(\frac{\tau_h}{\tau_h - \tau_l}\right)^{-1}\left(\frac{\tau_{hh} - \tau_l}{\tau_{hh}}\right)^{-1}$$

$$= \boxed{\left(\frac{\tau_h - \tau_l}{\tau_h}\right)\left(\frac{\tau_{hh}}{\tau_{hh} - \tau_l}\right)}$$

For $T_{hh} = 600K$, $T_l = 270K$, $T_h = 300K$,

$$\frac{Q_{hh}}{Q_h} = \frac{30}{300}\,\frac{600}{330} = \boxed{\frac{2}{11}}$$

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8.1(c).

Heat:

$$Q_{hh} = \sigma_{hh} \tau_{hh}.$$

$$W = Q_{hh}\left(1 - \frac{\tau_l}{\tau_{hh}}\right)$$

$$Q_h = \sigma_h \tau_h = Q_{l2} + W.$$

$$Q_l = \sigma_{hh} \tau_l.$$

$$Q_{l2} = \sigma_h \tau_l.$$

Entropy:

$$\sigma_{hh} = Q_{hh}/\tau_{hh}$$

$$\sigma_h = Q_h/\tau_h$$

(Notice the pump and carnot engine do not interact via entropy, because pure work carries no entropy).

$$\sigma_{hh} = Q_l/\tau_l$$

$$\sigma_h = Q_{l2}/\tau_l.$$